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MODEL STUDIES OF CURRENT COLLECTION BY A FLUSH MOUNTED DISK PROBE

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A simplified model of collection of charged particles by a flush mounted disk probe is presented. The model considers only collection of ions launched with a single fixed velocity normal to the collecting plane. It ignores collisions and space charge effects as well as the thermal distribution of the collected ions. The results of this model are compared with those of earlier, more detailed models, as well as with experimental results obtained from similar probes flown on rockets in the ionosphere.

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1. INTRODUCTION

In the introductory section of a review article on electrostatic probes, Chen[1] makes the following comment:

> "Experimentally, electrostatic probes are extremely simple devices, consisting merely of an insulated wire, using a D. C. power supply, and an ammeter or an oscilloscope. Nature, however, makes us pay a penalty for this simplicity: the theory of probes is extremely complicated."

The article which is introduced by this remark deals exclusively with probes described by a single spatial dimension; that is, probes assumed to be infinitely long cylinders, or those having spherical symmetry. Insofar as space vehicle probes are concerned, Chen's remark is even more true of flush mounted probes than of those he describes. Flush mounted probes are far simpler to construct, need not be deployed, and are more reliable and less costly than the necessarily boom mounted cylindrical or spherical probes. Their theory is also far more complicated, having at most cylindrical symmetry, and thus requiring at the least, two spatial dimensions for their proper description. As a result, we find that flush mounted probes continue to be flown on rockets and satellites, but that calculations are almost all numerical, limited in parameter range, and that interpretation of measurements still presents a serious problem.

Two studies of current collection for the front face of a right circular cylinder were carried out for AFGL in the 1970s, one by Parker[2] and one by Sugimura.[3] The former included the effects of vehicle velocity, disk potential, and space charge, while the latter dealt with all of these and also

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with the effects of collisions. In spite of the differences in methods of calculation, comparisons between the two studies can be made, and these reveal certain inconsistencies. There are also apparent disagreements between both of these studies and experimental flight data which are not understood. To aid in clearing up these problems, further studies and comparisons are thus indicated and it is for this purpose that the current effort was undertaken.

The present study is extremely simple, does not deal with either space charge or collisions, and in fact also neglects the angular aspects of the velocities associated with the thermal distribution of collected particles. It is reasonable to ask, given the far more realistic calculations already completed, whether such a model can be of any value at all. The answer to this question is in the affirmative. Simple models, whether in agreement with observations or the contrary, are always informative. In the usual course of events, simple models precede more complicated, complete ones, and questions of the utility of reversion from more complex to simpler do not arise. In the present instance, this does not seem to be the case, the more complete and complicated models having been explored before simpler ones. We therefore expect this study to be informative; and, these expectations will be seen to be justified.

The current collector for the model investigated here is a conducting, plane, circular disk at some specified potential, imbedded in and coplanar with an infinite conducting plane held at zero potential. Charged particles are released with a specified initial velocity toward and normal to the conducting plane, and at a distance great enough to simulate a release at infinity. The particles are assumed to move in the primary field of the conducting plane; that is, space charge effects are ignored. All particles have identical initial velocities so that there is no thermal distribution of initial energies. There is, however, an initial characteristic energy available for use in non-dimensionalizing the disk potential.

2. THE ELECTRIC FIELD

The potential for a conducting, conformal circular disk with unit potential on the inner (unit radius) disk, and zero potential on the surrounding conducting plane is given by

$$V(\rho, z) = \int_0^\infty J_0(\rho x) J_1(x) e^{-zx} dx, \quad (0 \le z \le \infty) \tag{1}$$

Here J_0 and J_1 are Bessel functions of order zero and one respectively, and ρ,z, the usual cylindrical coordinates. From this we have for the corresponding electric fields,

$$E_\rho = -\int_0^\infty x J_1(\rho x) J_1(x) e^{-zx} dx \quad (0 < z) \tag{2}$$

$$E_z = -\int_0^\infty x J_0(\rho x) J_1(x) e^{-zx} dx \quad (0 < z) \tag{3}$$

The approximate numerical evaluation of these integrals is time consuming, and to calculate a single trajectory for charged particles moving in this field may require thousands of field

evaluations. To mitigate this we proceed as follows. We break the half space $z \geq 0$ into two regions, an interior region, for which $\rho, z \leq 1.5$ and an exterior region for which either or both $\rho, z > 1.5$. In the interior region we evaluate the two field integrals at $30 \times 30 = 900$ discrete grid points, once and for all. As a particle trajectory enters this interior region, and field values are required at specific locations, these are supplied by a (linear) interpolation between the four bounding grid values. Higher accuracy could be obtained by using higher order interpolation but this was not, for present purposes, felt to be necessary. Although the potential is defined by the integral expression for $z = 0$, the fields are not: the integrals do not converge. We know from the boundary conditions however that $E_\rho(\rho,0) = 0$. For $E_z(\rho,0)$, we can calculate $V(\rho,\varepsilon)$, we know that $V(\rho,0) = 1$, and from these values, an approximate value of E_z at $z = 0$ can be obtained, as a numerical derivative. Alternatively, $E_z(\rho,0)$ may be extrapolated from the two contiguous values closest to $z = 0$.

For the exterior region, we make use of the following (unproved) theorem given by Smythe[4]: If V is symmetrical about the z axis and its value is known at all points on this axis and if this value can be expressed by a finite or convergent power series involving only integral powers of z, then the potential at any point can be obtained by multiplying the nth term of the power series by $P_n(\cos\theta)$ and writing r for z. The result holds for the same range of values of r, as the range of z in the original expression. $P_n(\cos\theta)$ is the Legendre Polynomial of order n.

Now, along the axis, the integral in Eq (1) reduces to

$$V = 1 - z \Big/ \sqrt{z^2+1} = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{1\cdot 3\cdot 5 \cdots 2n-1}{2^n\, n!\, z^{2n}} \tag{4}$$

the last valid for $z > 1$. Using the above theorem, we have for the potential in spherical coordinates

$$V(r,\theta) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{1\cdot 3\cdot 5 \cdots 2n-1}{2^n\, n!\, r^{2n}} P_{2n-1}(\cos\theta) \tag{5}$$

and, taking derivatives and using recursion relations for the Legendre polynomials,

$$E_r = -\frac{\partial V}{\partial r} = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{1\cdot 3\cdot 5 \cdots 2n-1}{2^{n-1}(n-1)!\, r^{2n+1}} P_{2n-1}(\cos\theta) \tag{6}$$

$$E_\theta = -\frac{1}{r}\frac{\partial V}{\partial \theta} = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{1\cdot 3\cdot 5 \cdots 2n-1}{2^n\, n!\, r^{2n+1}} \frac{1}{\sin\theta}\left[P_{2n-2}(\cos\theta) - \cos\theta\, P_{2n-1}(\cos\theta)\right] \tag{7}$$

Since trajectory calculations are best carried out in cylindrical coordinates, these values for E_r, E_θ are then converted to corresponding values for E_ρ, E_z.

The left hand side of Eq. (4) can also be expanded as a series in integral, positive powers of z, valid for $z < 1$. However, use of series for both $z > 1$ and $z < 1$ leaves values for $z \cong 1$ undetermined, a problem which is avoided by using integral evaluations for all $\rho, z \leq 1.5$.

The use of two widely differing methods of calculating the fields offers the opportunity to check on the accuracy of the numerical work by a comparison at the boundaries; and such a check shows the values to agree within better than 0.1 percent over the complete ranges of both boundaries.

The behavior of V, $\bar{E}$ at (1,0) remains to be discussed. We posit the nature of this behavior to be similar to that at the edge of an infinite strip at unit potential embedded in an infinite conducting plane at zero potential. The potential for this problem is readily found by use of complex variables, and for a strip of width 2 is found to be

$$V = \frac{1}{\pi}\tan^{-1}\left(\frac{2y}{x^2+y^2-1}\right) \tag{8}$$

where y is in the direction normal to, and x in the plane of, the strip. For the fields,

$$E_x = \frac{1}{\pi}\left(\frac{4xy}{(x^2+y^2-1)^2+4y^2}\right) \tag{9}$$

$$E_y = \frac{1}{\pi}\frac{-2(x^2-y^2-1)}{(x^2+y^2-1)^2+4y^2} \tag{10}$$

both of which are non-uniformly convergent at (±1,0).

From Eq. (10) the curve for which $E_y = 0$ is seen to be $x^2 - y^2 = 1$. The equivalent curve for the conformal disk, $E_z = 0$, is shown as the solid in Figure 1. For larger values of ρ,z the curve is given by $\rho = \sqrt{2}z$ which is the asymptotic value for a dipole field.

3. EQUATIONS AND RESULTS

The trajectory equations in cylindrical coordinates, in dimensional form are

$$m\frac{d^2\rho'}{dt^2} = eE'_{\rho'} \tag{11a}$$

$$m\frac{d^2z'}{dt^2} = eE'_{z'} \tag{11b}$$

with initial conditions

$$t = 0 \;:\; z' = z'_1 \;:\; \rho' = \rho'_1 \;:\; v'_{\rho_1} = 0 \;:\; v'_z = v'_{z_1}$$

From the last of these we have available an initial energy $\frac{1}{2}mV'^2_{z1}$ to utilize in non-dimensionalizing potentials and fields. We also set the unit of distance equal to the disk radius a, to obtain

$$\frac{d^2\rho}{d\tau^2} = \frac{\varphi}{2}E_\rho \tag{12a}$$

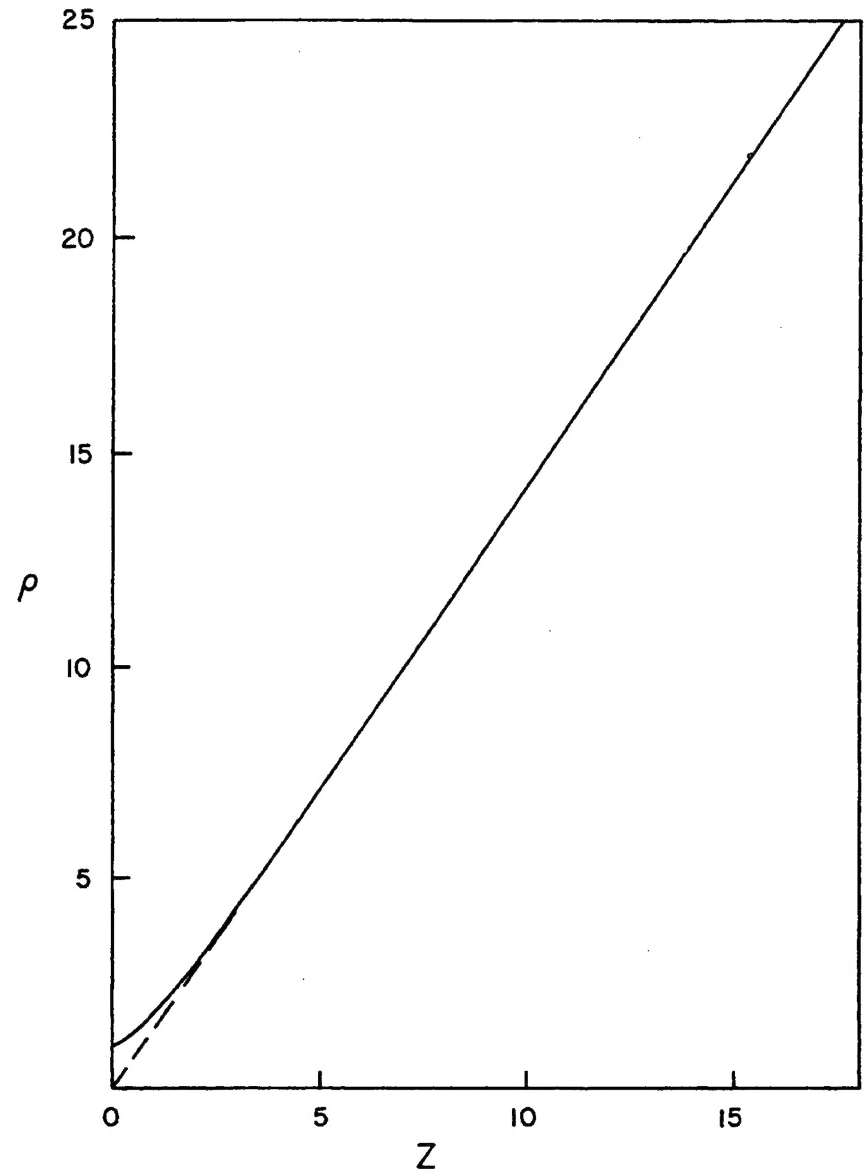

Figure 1. Curve of $E_z = 0$ for Disk-Plane Configuration

$$\frac{d^2z}{d\tau^2} = \varphi/2\, E_z \tag{12b}$$

$\tau = 0 \,:\, z = z_i \,:\, \rho = \rho_i \,:\, v_{\rho i} = 0 \,:\, v_{zi} = 1$ as the non-dimensionalized equations to be solved numerically. Here $\tau = tv'_{zi}/a,\ \rho_i = \rho'_i/a,\ z_i = z'_i/a;\ \varphi = \dfrac{eV'}{\frac{1}{2}mv'^2_{z_i}}$ is the non-dimensionalized disk potential, E_p, E_z are the non-dimensionalized fields for unit potential on the disk, and V' is the disk potential.

For an isolated charged disk, the field at infinity is that of a point charge, and it is possible to advance the particle from infinity to some fixed chosen location analytically before turning further advance over to a numerical procedure. For the conformal disk, the field at infinity is a dipole field, and there is no simple analytic solution for the trajectory. Hence the initial value of the z coordinate must be extended to rather large dimensionless values, somewhere between 50 and 200, the exact value depending on the applied potential. This is required to simulate charged particles incident from $z = \infty$.

Now, the object of this whole exercise is to find, for a given value of φ, that starting value (ρ_i, ∞) which terminates at $(\rho_f = 1, 0)$. In other words, we want to use a shoot and hunt method to try and hit a singular point of the force field, an endeavor for which the prospects of success are not very auspicious. But consider the following. Suppose that it turns out to be impossible to hit not only close to, but anywhere near the singularity. Suppose further, that for some value of initial $\rho = \rho_i$, $\rho_f << 1$, but that for $\rho = \rho_i + \varepsilon$, $\rho_f >> 1$. If ε can be made as small as desired, failure to converge on $\rho_f = 1$ is of no consequence. The collected current is given by a cross section, $\pi\rho_i^2$ and if ρ_i can be determined to within any desired accuracy, then so is the probe characteristic. As will be seen, this, in fact, will turn out to be the case.

The algorithm used to calculate trajectories was a fourth order Runge-Kutta routine. This allows ease of changing increment sizes, so that relatively larger increments can be used at the beginning of a trajectory, where both changes in field values and velocities are small, and smaller increments near the termination of trajectories where the opposite is true.

Figure 2 shows six complete trajectories, all for $\varphi = 200$, corresponding to six starting values for $\rho = \rho_i$. These trajectories are all in a constant θ plane, and negative values of ρ are those for which ρ is on the side of the axis opposite from its starting location. It is seen that for the five smaller values of ρ_i the final value of $\rho = \rho_f < 1$; that is, these particles are collected by the probe. For the value $\rho_i = 10.5$ on the other hand, $\rho_f = 1$ so that this particle is not collected. As remarked, we wish to discriminate between particles which are just collected, and those which are just not. Figure 3, again for $\varphi = 200$ illustrates this fine discrimination. The long dashed and solid trajectories, for $\rho_i = 9.90$ and 9.98 respectively, terminate at $\rho_f = 0.56$ and $\rho_f = 0.92$ and are hence collected, while the short dashed trajectory, with $\rho_i = 10.02$ terminates at $\rho_f = 4.0$ and is hence not collected. The jump in ρ_f is substantial; but that in ρ_i is small. If we choose the collection cross section radius to be 10.00, then this is given to within two parts in 10^4, a very good accuracy. Trajectories for other values of potentials are not shown, but they display the same fine discrimination in starting radii. It is notable that trajectories for values of ρ_i differing substantially from the transition value are relatively simple in form, while those close to the transition value are relatively complicated, showing reversals in sign for v_z and multiple transitions across the axis.

In Figure 4 are plotted values of ρ_f vs ρ_i for four values of potential. These clearly display the sharp transition from collected to uncollected particles. For the curves shown it can be seen that although closely approached, no particles terminate on the $z = 0$ plane for values of $\rho_f < -1$. This is also true for $\varphi = 300$ and 400 (not shown) and suggests the possibility that this is a general property of the trajectory system. Such a rule, if in fact true, cannot of course, be proved by numerical solutions.

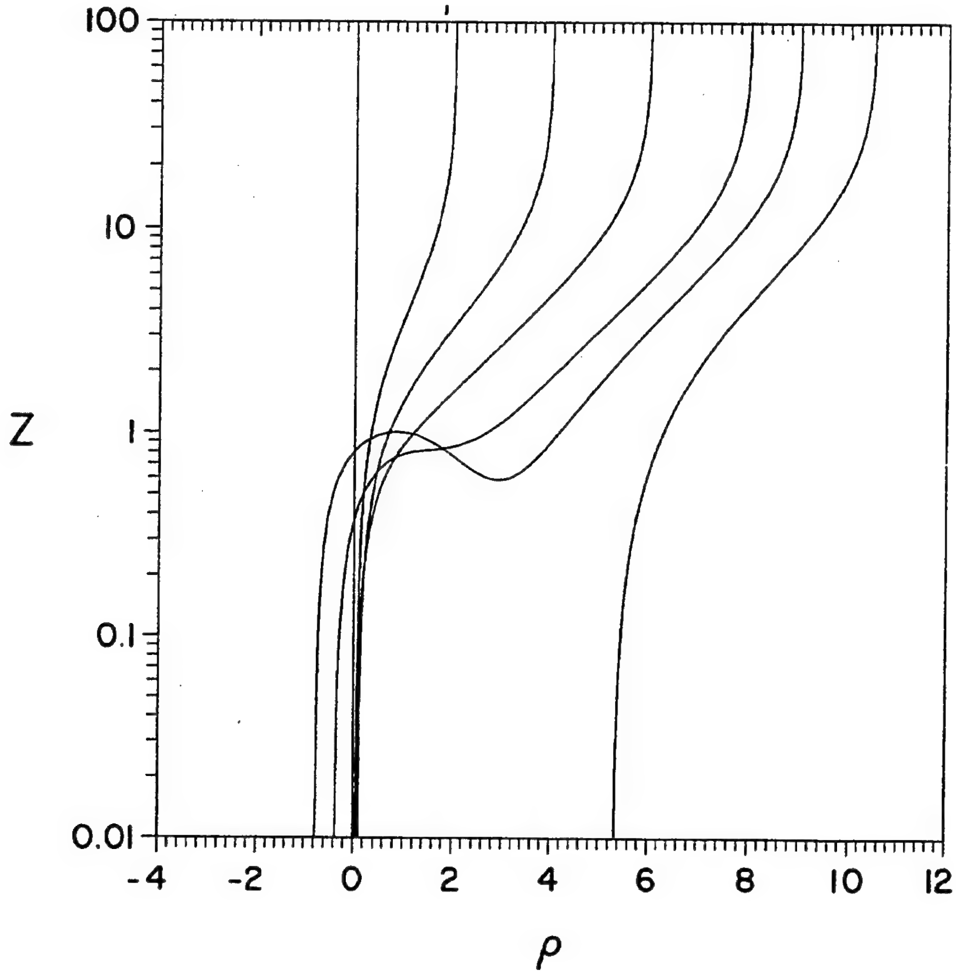

Figure 2. Typical Trajectories for $\varphi = 200$. $z_i = 100$

Finally, in Figure 5, we show the disk probe characteristic, the non-dimensionalized collected current, I, vs φ. Here, $I = \bar{\rho}_i^2(\varphi)$ where the bar indicates that value of ρ_i that separates $\rho_f < 1$ from $\rho_f > 1$. It is seen that the characteristic seems to be remarkably close to linear. This is discussed further in the following section.

4. DISCUSSION

4.1. Laplace Field

The calculated characteristic, closely approximated by $I = \alpha + \beta\varphi$ with $\alpha = 1.0$ and $\beta = 0.4875$ is based on an initial flux that is both monoenergetic and monodirectional. The first of these restrictions can be easily removed and doing this allows a re-nondimensionalization in terms of a temperature and thence, a meaningful comparison with other calculations of probe characteristics.

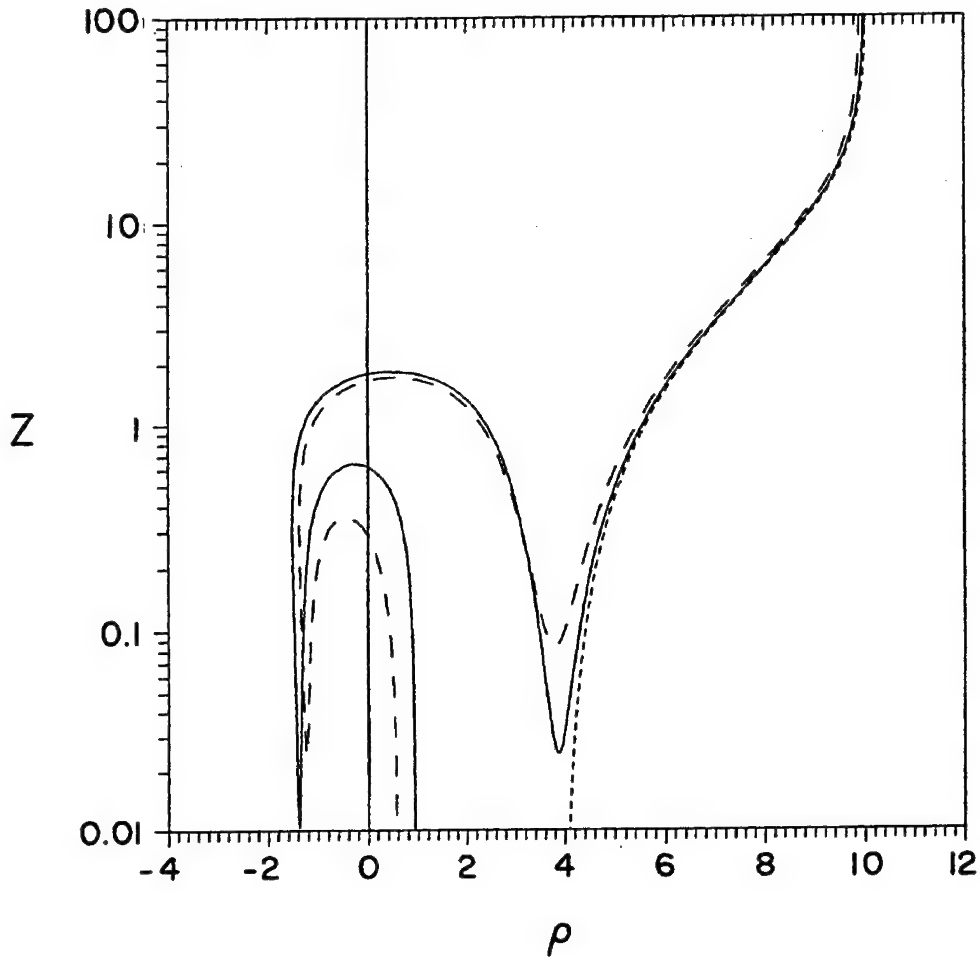

Figure 3. Trajectories Near Transition Values for $\varphi = 200$. Long Dash, $\rho_i = 9.9$; Solid, $\rho_i = 9.98$; Short dash, $\rho_i = 10.02$. $z_i = 100$.

The previous computation shows that for any fixed initial velocity, the characteristic is close to linear. If a characteristic is then formed by summing more than one initial velocity, it too will be nearly linear, and so, in fact will any characteristic formed from any distribution of initial velocities. If now, we consider a thermal distribution of such initial velocities, and replace the velocity square appearing in the denominator of φ with a mean square thermal velocity, we have

$$I = \alpha + \beta \frac{eV}{\frac{1}{2} m \langle v^2 \rangle} = \alpha + \frac{2}{3} \frac{eV}{kT} = \alpha + \frac{2}{3} \varphi' \equiv \alpha + \beta' \varphi' \tag{13}$$

Parker, (5), for a capped cylinder geometry with a Laplace field and no bulk flow also obtains (his Table 2) a nearly linear characteristic with $\beta' = 0.75$. Considering the difference in geometry as well as the radical differences in approach (he traces trajectories backwards in time, includes angles in distribution functions, computes the current densities only at the center of the disk, and uses a localized and relatively crude calculation grid) this agreement is impressive and reassuring as far as model predictions are concerned. Sugimura (3) with a capped geometry similar to

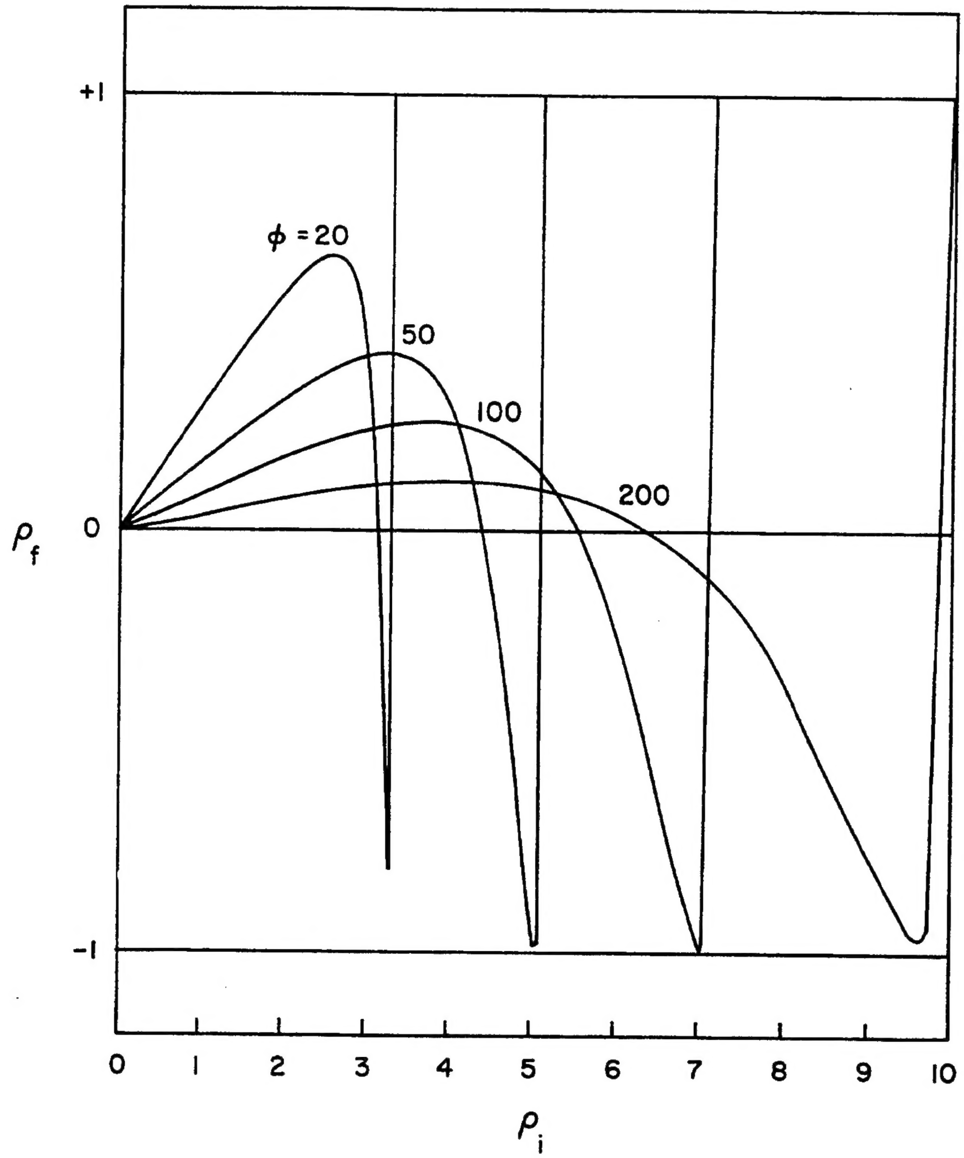

Figure 4. Four Curves Showing Final Radii ρ_f vs Initial Radii ρ_i. $z_i = 100$

Parker's, Laplace field, and zero bulk flow also calculates a voltage-current characteristic. (This value cannot be obtained directly but must be extracted from two separate figures in his report.) When this is done, even though his method of calculation differs radically from Parker's, a value close to unity for β' is again obtained.

4.2. Space Charge

The effect of space charge is in general to compress the electric field and thus reduce collected currents. Both Parker and Sugimura find that with this reduction, the approximate linearity of the current-voltage characteristics is maintained. The dimensionless parameter which characterizes

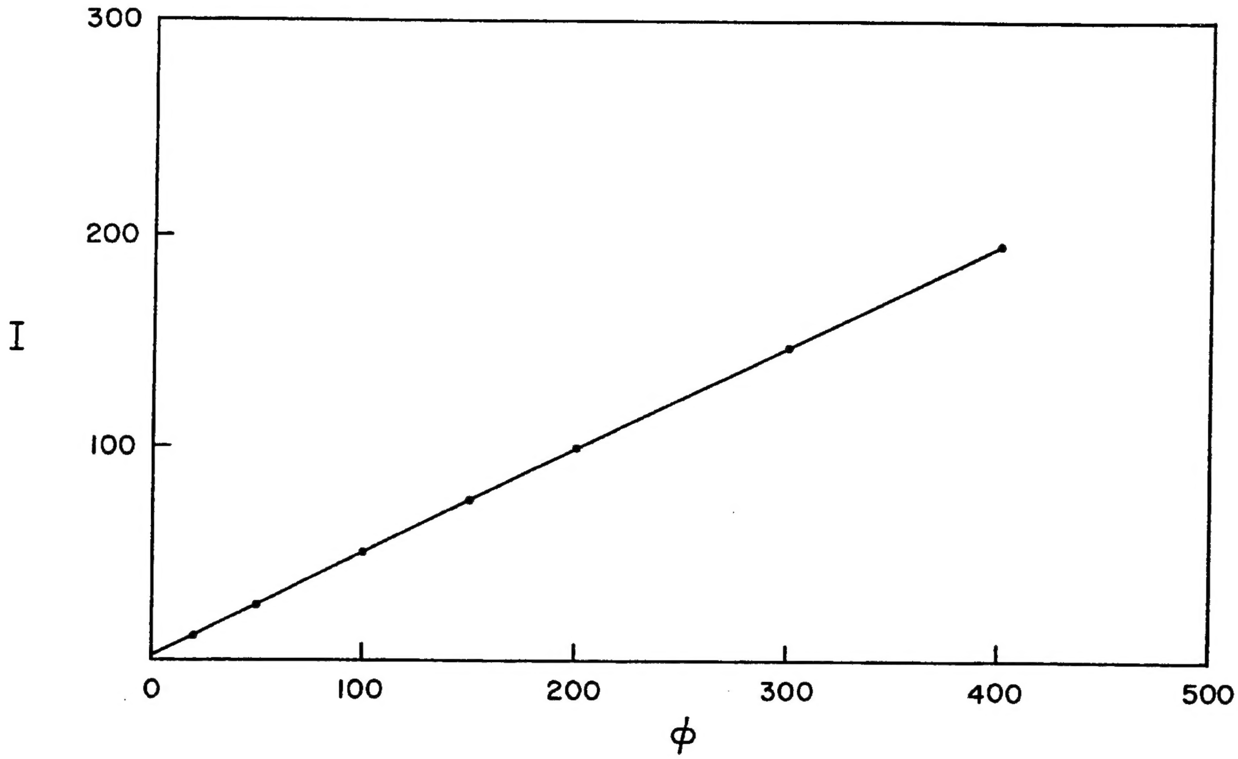

Figure 5. Collecting Characteristic for Disk-Plane Configuration. $z_i = 150$ for $\varphi = 300, 400$. I is the Current Normalized to $I_0 = \pi n e v_0$, that collected for $\varphi = 0$.

space charge effects is λ_D/D, the ratio of Debye length to collector disk diameter. Sugimura, whose studies include a broader range of parameter values finds that for λ_D/D greater than 0.3, there is essentially no space charge effect, while for λ_D/D of the order of 0.1, the reductions in current values are (depending somewhat on vehicle velocity) a factor of roughly between 2 and 3. Debye lengths in the ionosphere range from ~0.3 cm (high ion density near F_z peak) to greater than 1.0 cm in the lower E region. For a typical disk probe of ~1″ diameter then, we expect λ_D/D to range from a low of ~0.1 to a high of ~0.4. The corresponding reductions in slopes of characteristic curves would be a maximum of 3, and for the higher values of λ_D/D no reduction at all.

4.3. Comparison With Experimental Results

The disk probe data which are compared with the model are not from probes located along the sides of ionospheric measuring rockets, but are, rather, from the entrance aperture to a mass spectrometer. For the early ascent phase of the rocket trajectory the aperture plane is oriented close to normal to the ambient flow.

Aside from some indirect evidence involving highly speculative inferences, the comparison of model with measured characteristics requires the voltage on the collecting element to be swept. Unfortunately, for the location and geometry dealt with here, the data are scarce; as far as the author is aware, only one rocket flight yielding such data is available. These data have been discussed in a previous report by the author (6). The analysis in that report revealed a major

discrepancy between theory and data, and that analysis is still deemed valid. Whereas the model predicted ratios of currents at a collecting voltage of 10 volts to currents with no collecting voltage to be approximately 100, the data showed the measured ratio to be only approximately 4. The indirect evidence referred to above also indicates experimental values of current ratios far lower than the model ratios. The principal point being made here is that for a rather wide spectrum of models, embracing both different geometries as well as different methods of calculation, the model based characteristics are quite similar; and the differences between model predictions and measured current ratios are large. This point is strengthened considerably by the inclusion of the present simple model.

It would be convenient, as suggested by several workers in this field, to use conformal disk probes to measure RELATIVE values of charged particle densities in the ionosphere. Considering the large and currently unexplained differences between modeled and observed characteristics this would seem to be a questionable procedure. The agreement among the various model calculations lends confidence that the implications of the models have been correctly derived; the large differences between model predictions and measured results however, seem to indicate a major deficiency in the models themselves. Until this matter is resolved, caution is advisable in utilizing conformal probes to measure ambient charged particle densities.

4.4. Additional Comments

As previously noted, other workers have also found that a nearly linear characteristic is a common property of circular disk probes. The present calculations carry this result to an extreme and raise the question as to whether any computed deviations from linearity are the results of numerical errors, and that in fact, some property of the model leads to rigorously linear characteristics. This is an open question.

The great detail and extensive grid system used in the present work reveal a feature not seen in previous calculations of this type, namely the complexity of orbits for those particles released at radii near the transition values. The occurrence of such orbits raises an interesting question concerning their possible effect in calculations that include space charge. Continuity considerations require that the sharp spikes, having near zero velocities (Figure 3) be accompanied by high local charge densities. There is, as noted, a very limited range of initial values that contribute to such spikes. How this combination of few particles, but having relatively large space charge densities would affect overall space charge distributions and fields in a converged, iterated calculation is an interesting question.

References

1. Chen, Francis F. (1965) in *Plasma Diagnostic Techniques* Richard H. Huddlestone & Stanley L. Leonard, Eds, p. 114, Academic Press.

2. Parker, Lee W. (1970) *Theory of the External Sheath Structure and Ion Collection Characteristics of a Rocket-Borne Mass Spectrometer* Air Force Cambridge Laboratories Report AFCRL-71-0105 AD720833.

3. Sugimura, Takashi (1977) *Monte Carlo Simulation of Ion Collection by a Rocket-Borne Mass Spectrometer* Air Force Geophysics Laboratory Report AFGL-TR-77-0027 ADA039945.

4. Smythe, William R. (1950) *Static and Dynamic Electricity* 2nd Ed., p139, McGraw Hill.

5. Parker, Lee W. (1968) *Theory of the Circular Planar (Guardring) Langmuir Probe,* National Aeronautics and Space Administration report, contract NAS5-9088.

6. Sherman, Christopher (1972) *Ion Collection by a Positive Ion Mass Spectrometer*, Air Force Cambridge Research Laboratories report AFCRL-72-0038 AD740857.